Second Term Exam A.Y. 2021-22 (May-June 22)

Program : S.Y. B. Tech. Semester –IV

Course Code : 20UCC401B
Max. Marks: 60

Name of the Course : Engineering Statistics
Duration: 3 Hours

Instructions to the students
1. Each question carries 10 marks.
2 All questions are compulsory.
3. Illustrate your answers with neat sketches, diagram etc. wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Q1. Solve any Two questions.

a) A cyclist pedals from his house to his college at a speed of 10 m.p.h. and back from the college to his house at 15 m.p.h. Find the average speed. [5M]

b) A frequency distribution of heights (recorded to the nearest inch) of 100 male students at MGM University is given in the following Table. Find the standard deviation of the heights of the 100 male students at MGM University. [5M]

Height (in)	Number of Students
60–62	5
63–65	18
66–68	42
69–71	27
72–74	8

c) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Show that the mean is 7. Also find the other moments, $\beta 1$ and $\beta 2$. [5M]

Q2. Solve any Two questions.

a) The probability that a student will solve problem A is 2/3, and the probability that he will not solve problem B is 5/9. If the probability that student solves at least one problem is 4/5, what is the probability that he will solve both the problems? [5M]

b) In a bolt factory, machine A, B and C manufactured 25%, 35% and 40% bolt respectively of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the produced and is found to be defective. What is the probability that it was manufactured by machine B? [5M]

c) Four bad apples are mixed accidentally with 20 good apples. Obtain the probability distribution of the number of bad apples in a draw of 2 apples at random. [5M]

Q5. Solve any Two questions.

a) The numbers of scooter accidents per month in a certain town were as follows: [5M]

$$12, 8, 20, 2, 14, 10, 15, 6, 9, 4$$

Are these frequencies in agreement with the belief that accident conditions were the same during this 10 month period? (Table value of ψ^2 at 5% level for 9 d.f. is 16.919).

b) A machine is designed to produce insulating washers for electrical device of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262. [5M]

c) A stenographer claims that she can take dictation at the rate of 120 words per minute. Can we reject her claim on the basis of 100 trials in which she demonstrates a mean of 116 words with a standard deviation of 15 words? Use 5 percent level of significance. [Critical Value of Z at 5% level of significance is 1.96]. [5M]

Q6. Solve any Two questions.

a) Write the correct answer of the following: [5M]

i. The $\propto$ −cut set of any set is a …

(a) Crisp set (b) Fuzzy set (c) number set (d) none of these

ii. If $\tilde{A} = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.9)\}$ and $\propto = 0.3$ then $\propto_{\tilde{A}} = \ldots$

(a) {1,2,3} (b) {2,3,4} (c) {1,2,4} (d) none of these

iii. The characteristic of a crisp set may assigns a value …

(a) 1 (b) 0 (c) either 1 or 0 (d) none of these

iv. The membership function of a fuzzy set may assigns a value …

(a) 1 (b) 0 (c) any value of [0, 1] (d) none of these

v. If max $\mu_{\tilde{A}}(x) = 1$ for at least one $x \in X$, the set is …

(a) normal (b) convex (c) subnormal (d) none of these

b) Consider two fuzzy sets [5M]

$\underset{\sim}{A} = \left\{\frac{1}{2.0} + \frac{0.65}{4.0} + \frac{0.5}{6.0} + \frac{0.35}{8.0} + \frac{0}{10.0}\right\}$, $\underset{\sim}{B} = \left\{\frac{0}{2.0} + \frac{0.35}{4.0} + \frac{0.5}{6.0} + \frac{0.65}{8.0} + \frac{1}{10.0}\right\}$. Find the following:

i. $\underset{\sim}{A} \cup \underset{\sim}{B}$, $\underset{\sim}{A} \cap \underset{\sim}{B}$.

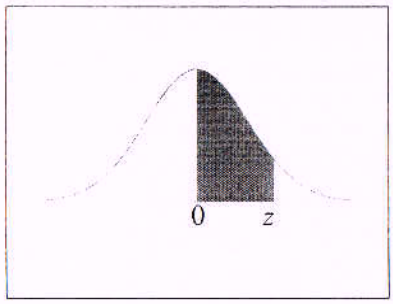
ii. $\overline{\underset{\sim}{A}}$, $\overline{\underset{\sim}{B}}$

c) Consider the fuzzy sets $\bar{A}, \bar{B}$ and $\bar{C}$ defined on the interval $x = [0, 10]$ of real numbers, by the membership grade function $\tilde{A} = \frac{x}{x+2}$, $\tilde{B} = \frac{1}{2^x}$, $\tilde{C} = \frac{1}{[1+10(x-2)^2]}$. Find the following: [5M]

i. $\widetilde{A^c}$, $\widetilde{B^c}$, $\widetilde{C^c}$

ii. $\tilde{A} \cup \tilde{B}$, $\tilde{B} \cup \tilde{C}$.

Standard Normal Distribution Table



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0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
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3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

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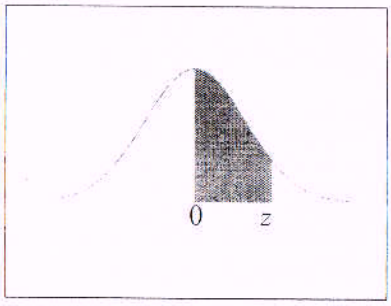

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3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

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Course Code : 20UCC401B

Max. Marks: 60

Name of the Course : Engineering Statistics

Duration: 3 Hours

Instructions to the students

1. Each question carries 10 marks.
2. All questions are compulsory.
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Q1. Solve any Two questions.

a) A cyclist pedals from his house to his college at a speed of 10 m.p.h. and back from the college to his house at 15 m.p.h. Find the average speed. [5M]

b) A frequency distribution of heights (recorded to the nearest inch) of 100 male students at MGM University is given in the following Table. Find the standard deviation of the heights of the 100 male students at MGM University. [5M]

Height (in)	Number of Students
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c) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Show that the mean is 7. Also find the other moments, $\beta 1$ and $\beta 2$. [5M]

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b) In a bolt factory, machine A, B and C manufactured 25%, 35% and 40% bolt respectively of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the produced and is found to be defective. What is the probability that it was manufactured by machine B? [5M]

c) Four bad apples are mixed accidentally with 20 good apples. Obtain the probability distribution of the number of bad apples in a draw of 2 apples at random. [5M]

Q3. Solve any Two questions.

a) The following is the frequency distribution of a number of heads turned up when 7 coins are tossed. [5M]

No. of heads	0	1	2	3	4	5	6	7
Frequency	7	6	19	35	30	23	7	1

i. Fit the binomial distribution for the data.
ii. Find expected frequencies.
iii. Find theoretical mean and standard deviation.

b) A car hire firm has two cars out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the probability of days on which car is not used and the proportion of days on which demand is refused. [5M]

c) A sample of 100 dry battery cells tested to find length of life produced the following results: [5M]

M = 12 hours, $\sigma = 3$ hours

Assuming the data to be normally distributed, what percentage of battery cells expected to have life

i. more than 15 hours
ii. less than 6 hours
iii. between 10 and 14 hours.

Q4. Solve any Two questions.

a) The ranks of the same 15 students in two subjects A and B are given below; the two numbers within the brackets denoting the rank of same student A and B respectively. [5M]

(1, 10), (2, 7), (3, 2), (4, 6), (5, 4), (6, 8), (7, 3), (8, 1), (9, 11), (10, 15), (11, 9), (12, 5), (13, 14), (14, 12), (15, 13).

Use Spearman's formula to find the rank correlation coefficient.

b) A simply supported beam carries a concentrated load P (kg), it's middle point corresponding to various values of P, the maximum deflection y cms is tabulated as: [5M]

P	100	120	140	160	180	200
y	0.90	1.10	1.20	1.40	1.60	1.70

Find a law of the form y = aP + b by using least square criteria.

c) From the data given below find: [5M]

i. The two regression coefficients.
ii. The two regression equations.
iii. The coefficient of correlation between the marks in Economics and Statistics.
iv. The most likely marks in Statistics when marks in Economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
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0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
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1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
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2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
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3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
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69–71	27
72–74	8

c) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Show that the mean is 7. Also find the other moments, $\beta 1$ and $\beta 2$. [5M]

Q2. Solve any Two questions.

a) The probability that a student will solve problem A is 2/3, and the probability that he will not solve problem B is 5/9. If the probability that student solves at least one problem is 4/5, what is the probability that he will solve both the problems? [5M]

b) In a bolt factory, machine A, B and C manufactured 25%, 35% and 40% bolt respectively of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the produced and is found to be defective. What is the probability that it was manufactured by machine B? [5M]

c) Four bad apples are mixed accidentally with 20 good apples. Obtain the probability distribution of the number of bad apples in a draw of 2 apples at random. [5M]

Q3. Solve any Two questions.

a) The following is the frequency distribution of a number of heads turned up when 7 coins are tossed. [5M]

No. of heads	0	1	2	3	4	5	6	7
Frequency	7	6	19	35	30	23	7	1

i. Fit the binomial distribution for the data.
ii. Find expected frequencies.
iii. Find theoretical mean and standard deviation.

b) A car hire firm has two cars out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the probability of days on which car is not used and the proportion of days on which demand is refused. [5M]

c) A sample of 100 dry battery cells tested to find length of life produced the following results: [5M]

M = 12 hours, σ = 3 hours

Assuming the data to be normally distributed, what percentage of battery cells expected to have life

i. more than 15 hours
ii. less than 6 hours
iii. between 10 and 14 hours.

Q4. Solve any Two questions.

a) The ranks of the same 15 students in two subjects A and B are given below; the two numbers within the brackets denoting the rank of same student A and B respectively. [5M]

(1, 10), (2, 7), (3, 2), (4, 6), (5, 4), (6, 8), (7, 3), (8, 1), (9, 11), (10, 15), (11, 9), (12, 5), (13, 14), (14, 12), (15, 13).

Use Spearman's formula to find the rank correlation coefficient.

b) A simply supported beam carries a concentrated load P (kg), it's middle point corresponding to various values of P, the maximum deflection y cms is tabulated as: [5M]

P	100	120	140	160	180	200
y	0.90	1.10	1.20	1.40	1.60	1.70

Find a law of the form y = aP + b by using least square criteria.

c) From the data given below find: [5M]

i. The two regression coefficients.
ii. The two regression equations.
iii. The coefficient of correlation between the marks in Economics and Statistics.
iv. The most likely marks in Statistics when marks in Economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

Q5. Solve any Two questions.

a) The numbers of scooter accidents per month in a certain town were as follows: [5M]

12, 8, 20, 2, 14, 10, 15, 6, 9, 4

Are these frequencies in agreement with the belief that accident conditions were the same during this 10 month period? (Table value of ψ^2 at 5% level for 9 d.f. is 16.919).

b) A machine is designed to produce insulating washers for electrical device of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262. [5M]

c) A stenographer claims that she can take dictation at the rate of 120 words per minute. Can we reject her claim on the basis of 100 trials in which she demonstrates a mean of 116 words with a standard deviation of 15 words? Use 5 percent level of significance. [Critical Value of Z at 5% level of significance is 1.96]. [5M]

Q6. Solve any Two questions.

a) Write the correct answer of the following: [5M]

i. The $\propto$ −cut set of any set is a …
(a) Crisp set (b) Fuzzy set (c) number set (d) none of these

ii. If $\tilde{A} = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.9)\}$ and $\propto = 0.3$ then $\propto_{\tilde{A}} = \ldots$
(a) {1,2,3} (b) {2,3,4} (c) {1,2,4} (d) none of these

iii. The characteristic of a crisp set may assigns a value …
(a) 1 (b) 0 (c) either 1 or 0 (d) none of these

iv. The membership function of a fuzzy set may assigns a value …
(a) 1 (b) 0 (c) any value of [0, 1] (d) none of these

v. If max $\mu_{\tilde{A}}(x) = 1$ for at least one $x \in X$, the set is …
(a) normal (b) convex (c) subnormal (d) none of these

b) Consider two fuzzy sets [5M]

$\underset{\sim}{A} = \left\{\frac{1}{2.0} + \frac{0.65}{4.0} + \frac{0.5}{6.0} + \frac{0.35}{8.0} + \frac{0}{10.0}\right\}$, $\underset{\sim}{B} = \left\{\frac{0}{2.0} + \frac{0.35}{4.0} + \frac{0.5}{6.0} + \frac{0.65}{8.0} + \frac{1}{10.0}\right\}$. Find the following:

i. $\underset{\sim}{A} \cup \underset{\sim}{B}$, $\underset{\sim}{A} \cap \underset{\sim}{B}$.
ii. $\overline{\underset{\sim}{A}}$, $\overline{\underset{\sim}{B}}$

c) Consider the fuzzy sets $\tilde{A}, \tilde{B}$ and $\tilde{C}$ defined on the interval $x = [0, 10]$ of real numbers, by the membership grade function $\tilde{A} = \frac{x}{x+2}$, $\tilde{B} = \frac{1}{2^x}$, $\tilde{C} = \frac{1}{[1+10(x-2)^2]}$. Find the following: [5M]

i. $\overline{\tilde{A}^c}$, $\overline{\tilde{B}^c}$, $\overline{\tilde{C}^c}$
ii. $\tilde{A} \cup \tilde{B}$, $\tilde{B} \cup \tilde{C}$.

MGM University
Aurangabad-431003
First Term Exam A.Y. 2021-22

Program : SY (All) | Sem –III
Course : Linear Algebra and Integral Transforms | Marks : 60
Course Code : 20UCC301B

Instructions to the students
1. Each question carries 10 marks.
2 All questions are compulsory
3. Illustrate your answers with neat sketches, diagram etc. wherever necessary
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Marks

Q1. Solve any two

a) Find the rank of the matrix: $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$. (5)

b) Find the values of λ and μ for which the system (5)

$$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$$

has: a unique solution, infinitely many solutions and no solution.

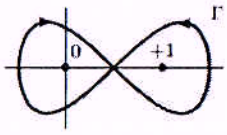
c) Find A^{-1} by using Cayley-Hamilton theorem when $A=\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (5)

Q2. Solve any two

a) Show that $f(z) = z + e^z$ is analytic and hence find its derivative. (5)

b) Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. (5)

c) Compute the contour integral $\int_\Gamma \frac{2z+1}{z(z-1)^2} dz$, where Γ is the contour depicted in the figure: (5)



Q3. Solve any two

a) Find the Laplace transform of $e^{2t}cos^2 t$. (5)

b) Express in terms of Heaviside unit step function and hence find its Laplace transform

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi/2 \\ \cos t, & t > \pi/2 \end{cases}$$ (5)

c) Use Laplace transforms technique to solve the initial value problem: (5)

$$y'' - 4y = e^{3t}, y(0) = 0, y'(0) = 0.$$

Q4. Solve any two

a) Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & x > 2 \end{cases}$ (5)

b) Find half range cosine series of $f(x) = x - x^2$ in (0, 1). (5)

c) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{\sin s}{s}\right) ds$. (5)

Q5. Solve any two

a) Find Z- transform of $sin3n\ sin5n$. (5)

b) Find Z- transform of $n\ sinan$. (5)

c) Solve f (n+2) +6f (n+1) +9f (n) = 2^n with f (0) = f (1) = 0 using z-transforms. (5)

Q6. Solve any two

a) Classify the following partial differential equations: (5)

(i) $2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 3\frac{\partial^2 u}{\partial x^2} = 0,$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

b) Use the method of separation of variables to solve the following partial differential equations:

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ when $u(x, 0) = 6e^{-3x}$. (5)

c) Find solutions of one dimensional heat equation by method of separation of variables. (5)

End of paper